

# Beam-size effect and particle losses at SuperB factory (Italy)

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## Abstract

In the colliders, the macroscopically large impact parameters give a substantial contribution to the standard cross section of the  $e^+e^- \rightarrow e^+e^-\gamma$  process. These impact parameters may be much larger than the transverse sizes of the colliding bunches. It means that the standard cross section of this process has to be substantially modified. In the present paper such a beam-size effect is calculated for bremsstrahlung at SuperB factory developed in Italy. We find out that this effect reduces beam losses due to bremsstrahlung by about 40%.

*Key words:* B-factories, beam-size effect, beam losses

*PACS:* 13.10.+q

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## 1 Introduction: beam-size or MD-effect

The so called beam-size or MD-effect is a phenomenon discovered in experiments [1] at the MD-1 detector (the VEPP-4 accelerator with  $e^+e^-$  colliding beams, Novosibirsk 1981). It was found that for ordinary bremsstrahlung, macroscopically large impact parameters should be taken into consideration. These impact parameters may be much larger than the transverse sizes of the interacting particle bunches. In that case, the standard calculations, which do not take into account this fact, will give incorrect results. The detailed description of the MD-effect can be found in review [2].

In the present paper we calculate the MD-effect and its influence on the beam particle losses at the SuperB factory developed in Italy [3]. We find out that this effect reduces beam losses due to bremsstrahlung by about 40%. For the reader convenience, we repeat briefly historical introduction from our paper [4].

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In 1980–1981 a dedicated study of the process  $e^+e^- \rightarrow e^+e^-\gamma$  has been performed at the collider VEPP-4 in Novosibirsk using the detector MD-1 for an energy of the electron and positron beams  $E_e = E_p = 1.8$  GeV and in a wide interval of the photon energy  $E_\gamma$  from 0.5 MeV to  $E_\gamma \approx E_e$ . It was observed [1] that the number of measured photons was smaller than expected. The deviation from the standard calculation reached 30% in the region of small photon energies and vanished for large energies of the photons. A qualitative explanation of the effect was given by Yu.A. Tikhonov [5], who pointed out that those impact parameters  $\varrho$ , which give an essential contribution to the standard cross section, reach values of  $\varrho_m \sim 5$  cm whereas the transverse size of the bunch is  $\sigma_\perp \sim 10^{-3}$  cm. The limitation of the impact parameters to values  $\varrho \lesssim \sigma_\perp$  is just the reason for the decreasing number of observed photons.

The first calculations of this effect have been performed in Refs. [6] and [7] using different versions of quasi-classical calculations in the region of large impact parameters. Later on, the effect of limited impact parameters was taken into account using the single bremsstrahlung reaction for measuring the luminosity at the VEPP-4 collider [8] and at the LEP-I collider [9].

A general scheme to calculate the finite beam size effect had been developed in paper [10] starting from the quantum description of collisions as an interaction of wave packets that form bunches. It has also been shown that similar effects have to be expected for several other reactions such as bremsstrahlung for colliding  $ep$ -beams [11], [12],  $e^+e^-$ -pair production in  $e^\pm e$  and  $\gamma e$  collisions [10].

In 1995 the MD-effect was experimentally observed at the electron-proton collider HERA [13] at the level predicted in [12].

It was realized in last years that the MD-effect in bremsstrahlung plays an important role in the beam lifetime problem. At storage rings TRISTAN and LEP-I, the process of single bremsstrahlung was the dominant mechanism for the particle losses in beams. If electron loses more than 1 % of its energy, it leaves the beam. Since the MD-effect considerably reduced the effective cross section of this process, the calculated beam lifetime in these storage rings was larger by about 25 % for TRISTAN [14] and by about 40 % for LEP-I [15] (in accordance with the experimental data) then without taken into account the MD-effect. According to our calculations [4], at B-factories PEP-II and KEKB the MD effect reduces beam losses due to bremsstrahlung by about 20 %.

In next Section we give the qualitative description of the MD-effect. In Sec. 3 we present our results for SuperB factory [3]. In the last Section we compare our results with those presented in Sec. 3.6.2 of the paper [3]. Though we find a good agreement we argue that this agreement is just a random coincidence because the basic ideas and formulas for these two results are quite different.

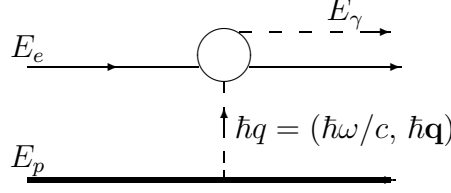


Fig. 1. Block diagram of radiation by the electron.

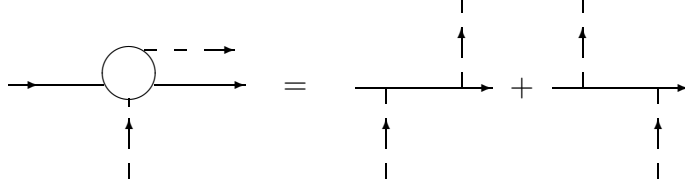


Fig. 2. Compton scattering of equivalent photon on the electron.

We present strong arguments in favor of our approach.

Below we use the following notations:  $E_e$  and  $E_p$  are the energies of the electron and positron,  $N_e$  and  $N_p$  are the numbers of electrons and positrons in the bunches,  $\sigma_H$  and  $\sigma_V$  are the horizontal and vertical transverse sizes of the bunch,  $r_e = e^2/(m_e c^2)$  is the classical electron radius,  $\gamma_e = E_e/(m_e c^2)$ ,  $\gamma_p = E_p/(m_p c^2)$  and  $\alpha \approx 1/137$ .

## 2 Qualitative description of the MD-effect

Qualitatively we describe the MD-effect using the  $ep \rightarrow ep\gamma$  process as an example. This reaction is described by the diagram of Fig. 1 which corresponds to the radiation of the photon by the electron (the contribution of the photon radiation by the proton can be neglected). The large impact parameters  $\varrho \gtrsim \sigma_\perp$ , where  $\sigma_\perp$  is the transverse beam size, correspond to small momentum transfer  $\hbar q_\perp \sim (\hbar/\varrho) \lesssim (\hbar/\sigma_\perp)$ . In this region, the given reaction can be represented as a Compton scattering (Fig. 2) of the equivalent photon, radiated by the proton, on the electron. The equivalent photons with frequency  $\omega$  form a “disk” of radius  $\varrho_m \sim \gamma_p c/\omega$  where  $\gamma_p = E_p/(m_p c^2)$  is the Lorentz-factor of the proton. Indeed, the electromagnetic field of the proton is  $\gamma_p$ -times contracted in the direction of motion. Therefore, at distance  $\varrho$  from the axis of motion a characteristic longitudinal length of a region occupied by the field can be estimated as  $\lambda \sim \varrho/\gamma_p$  which leads to the frequency  $\omega \sim c/\lambda \sim \gamma_p c/\varrho$ .

In the reference frame connected with the collider, the equivalent photon with energy  $\hbar\omega$  and the electron with energy  $E_e \gg \hbar\omega$  move toward each other (Fig. 3) and perform the Compton scattering. The characteristics of this pro-

cess are well known. The main contribution to the Compton scattering is given by the region where the scattered photons fly in a direction opposite to that of the initial photons. For such a backward scattering, the energy of the equivalent photon  $\hbar\omega$ , the energy of the final photon  $E_\gamma$ , and its emission angle  $\theta_\gamma$  are related by

$$\hbar\omega = \frac{E_\gamma}{4\gamma_e^2(1 - E_\gamma/E_e)} [1 + (\gamma_e\theta_\gamma)^2] \quad (1)$$

and, therefore, for the typical emission angles  $\theta_\gamma \lesssim 1/\gamma_e$  we have

$$\hbar\omega \sim \frac{E_\gamma}{4\gamma_e^2(1 - E_\gamma/E_e)}. \quad (2)$$

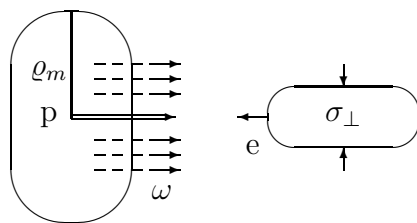


Fig. 3. Scattering of equivalent photons, forming the “disk” with radius  $\varrho_m$ , on the electron beam with radius  $\sigma_\perp$ .

As a result, we find the radius of the “disk” of equivalent photons with the frequency  $\omega$  (corresponding to a final photon with energy  $E_\gamma$ ) as follows:

$$\varrho_m = \frac{\gamma_p c}{\omega} \sim 4 \lambda_e \gamma_e \gamma_p \frac{E_e - E_\gamma}{E_\gamma}, \quad \lambda_e = \frac{\hbar}{m_e c} = 3.86 \cdot 10^{-11} \text{ cm}. \quad (3)$$

For the HERA collider with  $E_p = 820$  GeV and  $E_e = 28$  GeV one obtains

$$\varrho_m \gtrsim 1 \text{ cm} \quad \text{for} \quad E_\gamma \lesssim 0.2 \text{ GeV}. \quad (4)$$

Equation (3) is also valid for the  $e^-e^+ \rightarrow e^-e^+\gamma$  process with replacement the protons by the positrons. For the SuperB factory [3] it leads to

$$\varrho_m \gtrsim 1 \text{ cm} \quad \text{for} \quad E_\gamma \lesssim 0.1 \text{ GeV}. \quad (5)$$

The standard calculation corresponds to the interaction of the photons (that form the “disk”) with the unbounded flux of electrons. However, the particle beams at the HERA collider have finite transverse beam sizes of the order of  $\sigma_\perp \sim 10^{-2}$  cm. Therefore, the equivalent photons from the region  $\sigma_\perp \lesssim \varrho \lesssim \varrho_m$  cannot interact with the electrons from the other beam. This leads to the

reduction of the number of the observed photons. The “observed cross section”  $d\sigma_{\text{obs}}$  is smaller than the standard cross section  $d\sigma$  calculated for an infinite transverse extension of the electron beam,

$$d\sigma - d\sigma_{\text{obs}} = d\sigma_{\text{cor}}. \quad (6)$$

Here the correction  $d\sigma_{\text{cor}}$  can be presented in the form

$$d\sigma_{\text{cor}} = d\sigma_{\text{C}}(\omega, E_e, E_\gamma) dn(\omega) \quad (7)$$

where  $dn(\omega)$  denotes the number of “missing” equivalent photons and  $d\sigma_{\text{C}}$  is the cross section of the Compton scattering. Let us stress that the equivalent photon approximation in this region has a high accuracy (the neglected terms are of the order of  $1/\gamma_p$ ). But for the qualitative description it is sufficient to use the logarithmic approximation in which this number is (see[16], §99)

$$dn = \frac{\alpha}{\pi} \frac{d\omega}{\omega} \frac{dq_\perp^2}{q_\perp^2}. \quad (8)$$

Since  $q_\perp \sim 1/\varrho$ , we can present the number of “missing” equivalent photons in another form

$$dn = \frac{\alpha}{\pi} \frac{d\omega}{\omega} \frac{d\varrho^2}{\varrho^2} \quad (9)$$

with the integration region in  $\varrho$ :

$$\sigma_\perp \lesssim \varrho \lesssim \varrho_m = \frac{\gamma_p c}{\omega}. \quad (10)$$

As a result, this number is equal to

$$dn(\omega) = 2 \frac{\alpha}{\pi} \frac{d\omega}{\omega} \ln \frac{\varrho_m}{\sigma_\perp}, \quad (11)$$

and the correction to the standard cross section with logarithmic accuracy is (more exact expression is given by Eq. (17))

$$d\sigma_{\text{cor}} = \frac{16}{3} \alpha r_e^2 \frac{dy}{y} \left(1 - y + \frac{3}{4} y^2\right) \ln \frac{4\gamma_e \gamma_p (1 - y) \lambda_e}{y \sigma_\perp}, \quad y = \frac{E_\gamma}{E_e}. \quad (12)$$

### 3 MD-effect for SuperB factory

Usually in experiments the cross section is found as the ratio of the number of observed events per second  $d\dot{N}$  to the luminosity  $L$ . Also, in our case it is convenient to introduce the “observed cross section”, defined as the ratio

$$d\sigma_{\text{obs}} = \frac{d\dot{N}}{L}. \quad (13)$$

Contrary to the standard cross section  $d\sigma$ , the observed cross section  $d\sigma_{\text{obs}}$  depends on the parameters of the colliding beams. To indicate explicitly this dependence we introduce the “correction cross section”  $d\sigma_{\text{cor}}$  as the difference between  $d\sigma$  and  $d\sigma_{\text{obs}}$ :

$$d\sigma_{\text{obs}} = d\sigma - d\sigma_{\text{cor}}. \quad (14)$$

The relative magnitude of the MD-effect is given, therefore, by quantity

$$\delta = \frac{d\sigma_{\text{cor}}}{d\sigma}. \quad (15)$$

Let us consider the number of photons emitted by electrons in the process  $e^-e^+ \rightarrow e^-e^+\gamma$ . The standard cross section for this process is well known:

$$d\sigma^{(e)} = \frac{16}{3}\alpha r_e^2 \frac{dy}{y} \left(1 - y + \frac{3}{4}y^2\right) \left[ \ln \frac{4\gamma_e\gamma_p(1-y)}{y} - \frac{1}{2} \right], \quad y = \frac{E_\gamma}{E_e}, \quad (16)$$

where  $\gamma_e = E_e/(m_e c^2)$  and  $\gamma_p = E_p/(m_e c^2)$  is the Lorentz-factor for the electron and positron, respectively,  $\alpha = e^2/(\hbar c) \approx 1/137$  and  $r_e = e^2/(m_e c^2)$ .

The correction cross section depends on the r.m.s. transverse horizontal and transverse vertical bunch sizes  $\sigma_{jH}$  and  $\sigma_{jV}$  for the electron,  $j = e$ , and positron,  $j = p$ , beams. Besides, for the the considered collider we have to take into account that its  $e^\pm$  beams of the length  $l_e = l_p \equiv \sigma_z$  collide to a crossing angle  $2\psi$ . In calculations below we used data from Conceptual Design Report [3] (see Table 1).

Table 1: Parameters of beams used for calculations

$E_e$ , GeV	$E_p$ , GeV	$\sigma_V$ , $\mu\text{m}$	$\sigma_H$ , $\mu\text{m}$	$\sigma_z$ , cm	$2\psi$ , mrad	Energy spread, %
7	4	0.035	5.657	0.6	34	0.09

Formulas of the correction cross section for this case have been obtained in [11]. In the above notations the correction cross section is as follows:

$$d\sigma_{\text{cor}}^{(e)} = \frac{16}{3} \alpha r_e^2 \frac{dy}{y} \left[ \left( 1 - y + \frac{3}{4} y^2 \right) L_{\text{cor}} - \frac{1-y}{12} \right] \quad (17)$$

where

$$\begin{aligned} L_{\text{cor}} &= \ln \frac{2\sqrt{2}\gamma_e\gamma_p(1-y)(a_H + a_V)\lambda_e}{a_H a_V y} - \frac{3+C}{2}, \\ \lambda_e &= \frac{\hbar}{m_e c} = 3.86 \cdot 10^{-11} \text{ cm}, \quad C = 0.577\dots, \\ a_H &= \sqrt{\sigma_{eH}^2 + \sigma_{pH}^2 + (l_e^2 + l_p^2)\psi^2}, \quad a_V = \sqrt{\sigma_{eV}^2 + \sigma_{pV}^2}. \end{aligned} \quad (18)$$

The observed number of photons is smaller due to MD-effect than the number of photons calculated without this effect (Fig. 4).

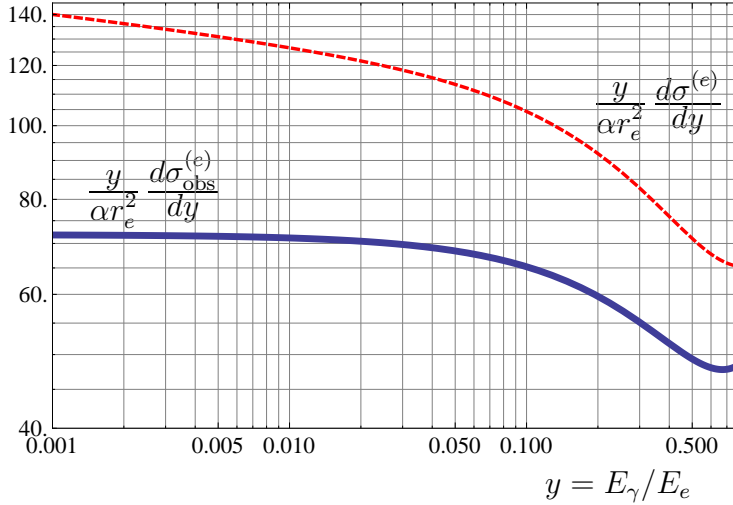


Fig. 4. The standard cross section  $(y/\alpha r_e^2)(d\sigma^{(e)}/dy)$  (the dashed curve) and the cross section with the beam-size correction  $(y/\alpha r_e^2)(d\sigma_{\text{obs}}^{(e)}/dy)$  (the solid curve) versus the relative photon energy  $y = E_\gamma/E_e$  for the SuperB factory

The relative magnitude of the MD-effect is given by quantity  $\delta$  from Eq. (15) (see Table 2). It can be seen from Fig. 4 and Table 2 that the MD-effect considerably reduces the differential cross section.

Table 2: Relative magnitude of the MD-effect for different photon energies

$y = E_\gamma/E_e$	0.001	0.005	0.01	0.05	0.1	0.5
$\delta$ , %	49	45	44	40	38	31

To estimate the integrated contribution of the discussed process into particle losses, we should integrate the differential observed cross section from some minimal photon energy. It is usually assumed that an electron leaves the bunch when it emits the photon with the energy either larger than 1 % of the electron energy or 10 times larger than the beam energy spread. In other words, the relative photon energy  $y = E_\gamma/E_e$  should be larger than  $y_{\min} = 0.01$  or  $y_{\min} = 0.009$ . In calculations below we use  $y_{\min} = 0.01$ . After integration of the differential observed cross section from  $y_{\min} \ll 1$  up to  $y_{\max} = 1$ , we obtain

$$\sigma_{\text{obs}}^{(e)} = \frac{16}{3} \alpha r_e^2 \left\{ \left( \ln \frac{1}{y_{\min}} - \frac{5}{8} \right) \left[ \ln \frac{\sqrt{2} a_H a_V}{(a_H + a_V) \lambda_e} + \frac{2 + C}{2} \right] + \frac{1}{12} \left( \ln \frac{1}{y_{\min}} - 1 \right) \right\} \quad (19)$$

or

$$\sigma_{\text{obs}}^{(e)}(y_{\min} = 0.01) = 166 \text{ mbarn} . \quad (20)$$

Let us note that the standard cross section integrated over the same interval of  $y$ , is equal to

$$\sigma^{(e)} = \frac{16}{3} \alpha r_e^2 \left\{ \left( \ln \frac{1}{y_{\min}} - \frac{5}{8} \right) \left[ \ln (4\gamma_e \gamma_p) - \frac{1}{2} \right] + \frac{1}{2} \left( \ln \frac{1}{y_{\min}} \right)^2 - \frac{3}{8} - \frac{\pi^2}{6} \right\} \quad (21)$$

or

$$\sigma^{(e)}(y_{\min} = 0.01) = 265 \text{ mbarn} . \quad (22)$$

Therefore, the observed cross section is smaller than the standard one by 37 %.

## 4 Discussion

In conclusion, we have calculated the MD-effect at the SuperB factory. We find out that this effect reduces beam particle losses due to bremsstrahlung by about 40%.

Then we compare our result (20) with that presented in Sec. 3.6.2 of the

paper [3]:

$$\sigma_{\text{obs}}^{(e)\text{CDR}}(y_{\text{min}} = 0.01) = 170 \text{ mbarn} . \quad (23)$$

We found out the good agreement between these two results. Unfortunately, this agreement is nothing else but a simple **random coincidence** because **the base of our approach and approach used in [3] is quite different**.

In a few words, the essence of our approach is the following. Those impact parameters  $\varrho$ , which give an essential contribution to the standard cross section at the discussed collider, reach values of  $\varrho_m \sim 2 \text{ cm}$  at  $E_\gamma = 0.01 E_e$  whereas the transverse size of the bunch is of the order of transverse bunch size  $\sigma_V$ . The limitation of the impact parameters to values

$$\varrho \lesssim \sigma_V = 0.035 \text{ } \mu\text{m} \quad (24)$$

is just the reason for the decreasing number of observed photons.

On the other hand, the results in CDR is based on BBBREM Monte simulation code of Ref. [17] which used a formula for the distance cut-off given in Ref. [18]. Authors of Refs. [17], [18] call this phenomenon as a density effect and used the cut-off at half the average distance  $d$  between two positrons in the bunch at rest (if we speak about emission by electrons). It correspond the limitation of the impact parameters to values

$$\varrho \lesssim d = \frac{1}{2} \left( \frac{\sigma_V \sigma_H \gamma_p \sigma_z}{N_p} \right)^{1/3} = 0.032 \text{ } \mu\text{m} . \quad (25)$$

The random coincidence of these two values is the origin of a good agreement between two results (20) and (23) . However, we should know which approach is correct, in order to understand tendencies in the case when some parameters of the collider will change.

We have a strong doubt about approach used in Refs. [17], [18]. From the theoretical point of view, we do not see any clear explanation, but a simple recipe. Besides, it contradicts to the existing HERA experiment [13].

On the contrary, our approach has a clear qualitative explanation given in Sec. 2. Our calculations are based on such a solid theory as QED and are confirmed by a number of experiments at the VEPP-4 collider in BINP (Novosibirsk) and at the HERA collider in DESY. In particular, in the VEPP-4 experiments [1], [5] and [8] it was studied not one but several different quantities, including the measurement of the effective cross section as function of the

transverse beam parameters (from  $\sigma_V = 10 \mu\text{m}$  to  $\sigma_V = 60 \mu\text{m}$ ) and dependence of the photon rate on the shift of one bunch in the vertical direction on the distance up to  $3\sigma_V$ . All these measurements supported the concept that the effect arises from the limitation of the impact parameters.

Certainly, accuracy of all experiments is far from excellent and further investigations are desirable, but from experimental point of view just now there is no another explanation with such a solid base as the MD-explanation.

## Acknowledgments

We are very grateful to A. Bondar, I. Koop and A. Onuchin for useful discussions. This work is partially supported by the Russian Foundation for Basic Research (code 09-02-00263) and by the Fond of Russian Scientific Schools (code 1027.2008.2).

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